

# Natural braneworld inflation in light of recent results from Planck and BICEP2

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In this paper we report on a major theoretical observation in cosmology. We present a concrete cosmological model for which inflation has natural beginning and natural ending. Inflation is driven by a cosine-form potential,  $V(\phi) = \Lambda^4 (1 - \cos(\phi/f))$ , which begins at  $\phi \lesssim \pi f$  and ends at  $\phi = \phi_{\text{end}} \lesssim 5f/3$ . The distance traversed by the inflaton field  $\phi$  is sub-Planckian. The Gauss-Bonnet term  $\mathcal{R}^2$  arising as leading curvature corrections in the action  $S = \int d^5x \sqrt{-g_5} M^3 (-6\lambda M^2 + R + \alpha M^{-2} \mathcal{R}^2) + \int d^4x \sqrt{-g_4} (\dot{\phi}^2/2 - V(\phi) - \sigma + \mathcal{L}_{\text{matter}})$  (where  $\alpha$  and  $\lambda$  are constants and  $M$  is the five-dimensional Planck mass) plays a key role to terminate inflation. The model generates appropriate tensor-to-scalar ratio  $r$  and inflationary perturbations that are consistent with *Planck* and BICEP2 data. For example, for  $N_* = 50 - 60$  and  $n_s \sim 0.960 \pm 0.005$ , the model predicts that  $M \sim 5.64 \times 10^{16}$  GeV and  $r \sim (0.14 - 0.21)$  [ $N_*$  is the number of e-folds of inflation and  $n_s(n_t)$  is the scalar (tensor) spectrum spectral index]. The ratio  $-n_t/r$  is (13% – 24%) less than its value in 4D Einstein gravity,  $-n_t/r = 1/8$ . The upper bound on the energy scale of inflation  $V^{1/4} = 2.37 \times 10^{16}$  GeV ( $r < 0.27$ ) implies that  $(-\lambda\alpha) \gtrsim 75 \times 10^{-5}$  and  $\Lambda < 2.17 \times 10^{16}$  GeV, which thereby rule out the case  $\alpha = 0$  (Randall-Sundrum model). The true nature of gravity is holographic as implied by the braneworld realization of string and M theory. The model correctly predicts a late-epoch cosmic acceleration with the dark energy equation of state  $w_{DE} \approx -1$ .

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## I. INTRODUCTION

Cosmic inflation [1, 2] – the hypothesis that the Universe underwent a rapid exponential expansion in a brief period following the big bang – is a theoretically attractive paradigm for explaining many problems of standard big-bang cosmology, including why the Universe has the structure we see today [3, 4] and why it is so big. It could also solve outstanding puzzles of standard big-bang cosmology, such as, why the Universe is, to a very good approximation, flat and isotropic on largest scales.

To get a successful inflationary model that respects various observational constraints from the *Wilkinson Microwave Anisotropy Probe* (WMAP) [5], *Planck* [6] and *Background Imaging of Cosmic Extragalactic Polarization* (BICEP2) [7] and other experiments, namely, those related to the cosmic microwave background (CMB) observations, it is necessary to obtain an inflationary potential  $V(\phi)$  having the height  $V^{1/4}$  much smaller than its width  $\Delta\phi$  (the distance traversed by the  $\phi$ -field during inflation). Moreover, observational results from *Planck* [6] and BICEP2 [7] lead to an upper bound on the energy scale of inflation,  $V_*^{1/4} = 1.94 \times 10^{16}$  GeV ( $r_*/0.12$ ), where  $r_*$  is the (maximum) ratio of tensor-to-scalar fluctuations of the primordial power spectra, while ideas based on fundamental theories of gravity, such as, superstring and supergravity [8–10], reveal that  $\Delta\phi \sim M_P$  (where  $M_P = 2.43 \times 10^{18}$  GeV is the reduced Planck mass). These two very different mass scales (differing by at least 2 orders of magnitude) is what is known as the fine-tuning problem in inflation. The latter usually requires precise couplings in the theory so as to prevent radiative corrections from bringing the two mass scales back to the same level. An inflationary model parametrized by the following cosine-type poten-

tial [8, 11, 12]:

$$V(\phi) = \Lambda^4 \left( 1 \pm \cos \left( \frac{\phi}{f} \right) \right), \quad (1.1)$$

where  $\Lambda \sim m_G$  is the vacuum expectation value of the grand unified theory (GUT) Higgs fields, or the symmetry breaking mass scale of the GUT  $\sim (1-2) \times 10^{16}$  GeV, avoids this problem mainly because it uses shift symmetries  $\phi = \phi \pm 2\pi f$  to generate a flat potential, which is protected from radiative corrections in a natural way [12]. The above potential represents a potential of pseudo Nambu-Goldstone boson of the grand unified theory, which was initially motivated by its origin in symmetry breaking in an attempt to naturally give rise to the extremely flat potentials required for inflationary cosmology, known as the natural inflation model [11].

In string theory and some super-gravity models, the effective scale  $\Lambda$  is small compared to  $f$  due to the exponential (instanton) suppression factor, such as,  $\Lambda = \alpha_0 e^{-\alpha_1 f}$  and  $\alpha_1 \gg \ln \alpha_0 > 0$  [12–14]. A cosine-form potential as in (1.1) is obtained also in particle physics models with weakly self-coupled (pseudo-)scalars, such as, the axion [15] and the extra component of a gauge field in a 5D theory compactified on a circle [16–18]. In the limit of exact symmetry (e.g., supersymmetry),  $\phi$  is a flat direction, so some tilt is necessary for cosmic inflation. This is provided by explicit symmetry breaking terms, which can be mediated, for example, by gravitational quantum corrections. It is thus natural to include the leading-order curvature corrections also in a gravitational Lagrangian.

For the sake of convenience we define  $\Lambda^4 \equiv V_0$ , so that

$$V(\phi) = V_0 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right]. \quad (1.2)$$

We have taken the negative sign in (1.1) so that  $\phi = 0$  is the true minimum. It is straightforward to obtain

$$V_\phi^2 = \frac{V}{f^2} (2V_0 - V), \quad V_{\phi\phi} = \frac{1}{f^2} (V_0 - V). \quad (1.3)$$

For  $\phi \ll f$ ,  $V(\phi)$  gives an approximately quadratic potential,  $V(\phi) = m^2 \phi^2$  [with  $m^2 \equiv \Lambda^4/(2f^2)$ ], which was studied in [19] in the context of braneworld inflation. In this limit  $V_\phi^2 = 4Vm^2$  and  $V_{\phi\phi} = 2m^2$ . Here we work in a general scenario where  $\phi$  is unconstrained. Recently, in [20], it was argued that the natural inflation model first proposed in [11] and the so-called *extranatural* inflation model [18] can have distinguishing inflationary signatures.

The Planck collaboration and some earlier discussions showed that in Einstein gravity the potential (1.2) leads to results compatible with Planck data for inflation if  $f \gtrsim (15/\pi)M_P$  in the large field limit. The assumption that the inflaton field  $\phi$  may take values larger than the Planck scale and/or it traverses a distance large compared with the Planck mass during inflation is outside the range of validity of an effective field theory description, so it is natural to assume that  $f \lesssim M_P$ . In this paper we show that  $\mathcal{R}^2$ -type curvature corrections in a 5D Lagrangian can remove this drawback of the original natural inflation model, giving a trustworthy model of inflation.

Another important physical quantity that we would need for studying impacts of the above mentioned potential on the primordial scalar and tensor fluctuations is the Hubble expansion parameter. Einstein gravity often fails to explain the high energy phenomena. Moreover, according to some fundamental theories of gravity and particle interactions, including superstring theory, the true nature of gravity is higher dimensional, whereas the elementary particles, fundamental scalars, and gauge fields of the standard quantum field theory live within a four-dimensional (three dimensions of space and one dimension of time) membrane, or “brane”. This idea is consistent with gravity/gauge-theory correspondence [21, 22], which provides so far the best understanding of string theory in terms of gauge field theories, such as, the Yang-Mills theory.

As the most natural generalization of Einstein gravity in five dimensions, we consider the following action [23]

$$\begin{aligned} S &= S_{\text{bulk}} + S_{\text{brane}} \\ &= \int_{\mathcal{M}} d^5x \sqrt{|g|} M^3 \left( -6\lambda M^2 + R + \frac{\alpha}{M^2} \mathcal{R}^2 \right) \\ &\quad + \int_{\partial\mathcal{M}} d^4x \sqrt{|\tilde{g}|} (-\sigma + \mathcal{L}_\phi + \mathcal{L}_{\text{matter}}), \end{aligned} \quad (1.4)$$

where  $\alpha$  and  $\lambda$  are constants,  $\sigma$  is the brane tension,  $M$  is the five-dimensional Planck mass,  $R$  is the Einstein-Hilbert term,  $\mathcal{R}^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$  is the Gauss-Bonnet (GB) density and  $\mathcal{L}_\phi = \dot{\phi}^2/2 - V(\phi)$  is the scalar Lagrangian. The GB density, which appears in the low energy effective action of heterotic string theory and in Calabi-Yau compactifications of M theory, is known to give solutions that are free of ghosts about flat and other exact backgrounds, such as a

warped spacetime background [24]. The  $\mathcal{R}^2$  terms can arise as the  $1/\mathcal{N}$  corrections in the large  $\mathcal{N}$  limit of some gauge theories and thus provide a testing ground to investigate the effects of higher-curvature terms in the context AdS/CFT correspondence [25, 26]. In this paper we discuss the wider cosmological implications of the theory.

The brane action (also known as boundary action) is crucial to obtain the correct form of Friedman equations in four dimensions. The matter Lagrangian  $\mathcal{L}_m$  can be ignored at sufficiently high energy,  $V^{1/4} \gtrsim 10^{15}$  GeV. For a cosine-form potential given above, inflation begins once the inflaton field  $\phi$  is displaced from  $\phi = \pi f$ , possibly breaking a fundamental symmetry of the GUT potential. If the bulk spacetime is negatively curved [anti-de Sitter (AdS)]  $\lambda < 0$  and the GB coupling  $\alpha > 0$ , then inflation would have a natural end. Because of this reason the present model may be viewed as a “doubly natural inflation” scenario.

The recent detection of a gravitational wave contribution to the CMBR anisotropy by BICEP2 [7] with a relatively large tensor-to-scalar ratio  $r \sim 0.19 (+0.007 - 0.005)$  may be viewed as a clear cosmological gravitational wave signature of inflation [27]. By reanalyzing the BICEP2 results, the authors of Ref. [28] argued that BICEP2 data are consistent with a cosmology with  $r = 0.2$  and negligible foregrounds, but also with a cosmology with  $r = 0$  and a significant dust polarization signal (see, e.g., [29, 30] for some other implications of BICEP2 results). This ambiguity may be resolved by future Keck Array observations at 100 GHz and Planck observations at higher frequencies.

One of the motivations for considering the effects of  $\mathcal{R}^2$  terms on inflationary scalar and tensor perturbation amplitudes is to obtain a relatively large  $r$  that is compatible with the BICEP2 result, namely,  $r = 0.19^{+0.07}_{-0.05}$  (or  $r = 0.16^{+0.06}_{-0.05}$  after subtracting an estimated foreground). The value of  $r$  reported by BICEP2 is larger than the bounds  $r < 0.13$  and  $r < 0.11$  reported by WMAP [5] and Planck [6]. Many authors have considered various possibilities [31–35] for the origin of a cosmological gravitational wave signature that support a value  $r > 0.11$ . If the B-mode polarization detected by BICEP2 is due to primordial gravitational waves, then it implies that inflation was driven by energy densities at the GUT scale  $m_{\text{GUT}} \sim \Lambda \sim (1 - 2) \times 10^{16}$  GeV. The results in this paper support this idea.

## II. INFLATIONARY PARAMETERS

The Hubble expansion parameter in four dimensions is uniquely given by [19, 36, 37]

$$H^2 = \frac{M^2 \psi^2}{|\beta|} [(1 - \beta) \cosh \varphi - 1], \quad (2.1)$$

where

$$\varphi \equiv \frac{2}{3} \sinh^{-1} \left[ \frac{\rho_\phi + \sigma}{\psi M^4} \frac{|2\beta|^{1/2}}{4(1 - \beta)^{3/2}} \right], \quad (2.2)$$

and

$$\beta \equiv 4\alpha\psi^2 = 1 \pm \left(1 + 8\lambda\alpha + \frac{8\alpha\mathcal{E}}{a^4M^2}\right)^{1/2}, \quad (2.3)$$

where  $a$  is the scale factor of the physical universe. We will take the negative root which has a smooth Einstein gravity or Randall-Sundrum limit ( $\alpha = 0$ ) [38].  $\mathcal{E}$  is a measure of bulk radiation energy, which is proportional to the mass of a 5D black hole and  $\psi$  is a dimensionless measure of bulk curvature ( $\psi > 0$  for an anti-de Sitter bulk and  $\psi < 0$  for de Sitter bulk; the  $\psi = 0$  case which corresponds to a flat 5D Minkowski spacetime must be treated separately). There are three bulk parameters here:  $\alpha$ ,  $\lambda$ , and  $\beta$ .  $\alpha$  and  $\lambda$  are taken to be constants, while  $\beta$  would vary with the evolution of the Universe, especially after reheating since  $\mathcal{E} > 0$ . During inflation (more specifically, before reheating)  $\beta$  is also a constant since  $\mathcal{E} \approx 0$  (as there is no radiation energy or at least not in an appreciable amount) and the scale factor rapidly grows. The choice  $\beta < 0$  is also possible, provided that the bulk is de Sitter ( $\lambda > 0$ ), but we will not study this case here as it does not lead to a graceful exit from inflation.

In the original braneworld proposal [38] the 3-brane tension  $\sigma$  is assumed to be a constant (which is forced upon only in the static limit where the Hubble expansion parameter is zero). In an expanding physical universe, the brane tension can be a function of the four-dimensional scale factor  $a(t)$ , or the volume of the Universe. All the results in this paper are valid even if  $\sigma$  is scale dependent ( $\sigma = \sigma(a)$ ), as long as the condition  $\rho_\phi \gg \sigma$  holds during inflation.

As is usually the case, the Hubble expansion parameter is linked to the four-dimensional scalar-matter density  $\rho_\phi \equiv \frac{1}{2}\dot{\phi}^2 + V(\phi)$ . Further, from Eq. (2.2) we can see that  $\rho_\phi$  can be defined in terms of the energy scale  $\varphi$ , which is dimensionless. This is a direct manifestation of holography or gravity/gauge-theory correspondence. The inflaton equation of motion is

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + V_\phi = 0, \quad (2.4)$$

where  $V_\phi \equiv dV/d\phi$ . Under the slow-roll approximation  $\ddot{\phi} \ll 3H(t)\dot{\phi}$ ,  $\rho_\phi \simeq V \gg \sigma$ ,\* we get

$$V \simeq \frac{4(1-\beta)^{3/2}}{(2\beta)^{1/2}} \psi M^4 \sinh(3\varphi/2). \quad (2.5)$$

By substituting this expression in Eq. (1.3) we obtain the slow-roll parameters:

$$\begin{aligned} \epsilon &= -\frac{\dot{H}}{H^2} \simeq \frac{dH}{d\varphi} \frac{d\varphi}{dV} \frac{V_\phi^2}{3H^3} \\ &= \frac{2\beta(1-\beta)V_0}{9M^2\psi^2f^2} \frac{\sinh\varphi \tanh(3\varphi/2)(1-X(\varphi))}{[(1-\beta)\cosh\varphi - 1]^2}, \end{aligned} \quad (2.6)$$

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\* The brane tension can be neglected for inflationary calculations; it finds an interesting role at low energies, such as during a transition from the decelerating to accelerating phase.

$$\eta = \frac{V_{\phi\phi}}{3H^2} = \frac{\beta V_0}{3M^2\psi^2f^2} \frac{(1-2X(\varphi))}{[(1-\beta)\cosh\varphi - 1]}, \quad (2.7)$$

where

$$X(\varphi) \equiv (1-\beta)^{3/2} \sinh\left(\frac{3\varphi}{2}\right) \chi, \quad \chi \equiv \frac{M^4}{\sqrt{2\alpha}V_0}.$$

Typically,  $V_0 \sim M^4$  and  $\alpha \gtrsim 10^3$ , so  $\chi < 0.05$ . After a few e-folds of cosmic inflation,  $X \ll 1$  and  $V(\phi)$  approximates to the quadratic  $m^2\phi^2$  potential. As we establish below, the model deviates from the GB assisted  $m^2\phi^2$ -inflation [19], especially, at higher energies ( $\varphi \gtrsim 1$ ).

We will work under the assumption that  $\beta \ll 1$  and inflation ends at  $\varphi = \varphi_e \ll 1$ , where subscript ‘e’ refers to the end of inflation. We will justify these assumptions. Inflation ends ( $\epsilon \geq 1$ ) at  $\varphi = \varphi_e$  when

$$\frac{2\beta(1-\beta)V_0}{9M^2\psi^2f^2} \simeq \frac{\varphi_e^2}{(6-8\beta)}. \quad (2.8)$$

The second term above can be expressed also in terms of the number of e-folds of inflation,  $N \equiv \int H dt$ ; the number of e-folds is well approximated by

$$N \equiv \int_{\varphi_*}^{\varphi_e} H \frac{dt}{d\phi} \frac{d\phi}{dV} \frac{dV}{d\varphi} d\varphi \simeq 3 \int_{\varphi_*}^{\varphi_e} \frac{H^2}{V_\phi^2} \left(\frac{dV}{d\varphi}\right) d\varphi,$$

where the equality holds in the slow-roll approximation  $\ddot{\phi} \ll 3H(t)\dot{\phi}$ .  $\varphi_*$  is the value of  $\varphi$  where the mode  $k_* = a_*H_*$  crosses the Hubble radius for the first time (during inflation). We will assume that  $\chi \sinh(3\varphi_*/2) < 1$ . As a good approximation we have

$$N_* \simeq \frac{9M^2\psi^2f^2}{4\beta V_0} \left( I(\varphi_*) - \frac{(2-5\beta)\varphi_e^2}{12} \right), \quad (2.9)$$

where the function  $I(\varphi)$  is well approximated by

$$\begin{aligned} I(\varphi) &= \varphi - \frac{2\beta}{3} \ln(e^\varphi - 1) + (1-\beta)(\cosh\varphi - 1) \\ &\quad + \frac{3-\beta}{3} [\ln 3 - \ln(e^{2\varphi} + e^\varphi + 1)] \end{aligned} \quad (2.10)$$

From Eqs. (2.8) and (2.9) we establish that

$$\frac{2\beta(1-\beta)V_0}{9M^2\psi^2f^2} \simeq \frac{\varphi_e^2}{(6-8\beta)} \simeq \frac{I(\varphi_*)(1-\beta)}{2N_* + 1 - 5\beta}. \quad (2.11)$$

We can drop the term  $5\beta$  since the number of e-folds required to explain the flatness and horizon problems of the hot big-bang cosmology is large,  $N_* \sim 50 - 62$ , depending upon the detail of reheating mechanism after the end of inflation, while  $|\beta| \ll 1$ . The above matching condition works well for  $\beta \lesssim 10^{-2}$ . To a good approximation,

$$\epsilon = \frac{(1-\beta)I(\varphi_*)}{2N_* + 1} \frac{\sinh\varphi_* \tanh(3\varphi_*/2)(1-X(\varphi_*))}{[(1-\beta)\cosh\varphi_* - 1]^2}, \quad (2.12)$$

$$\eta = \frac{3I(\varphi_*)}{2(2N_* + 1)} \frac{(1-2X(\varphi_*))}{(1-\beta)\cosh\varphi_* - 1}. \quad (2.13)$$

In the discussion below we take  $\varphi_* < 2.5$ , so that  $X(\varphi_*) < 0.3$ . As shown in Fig. 1, inflation has a natural exit ( $\epsilon > 1$ ) only if  $\beta > 0$ , which means  $\lambda < 0$ . The Planck+WP (or WMAP 9-yr large angular scale polarization) constraints imply  $\epsilon < 0.01$  and  $\eta < 0.008$  at 95CL. This result is fully compatible with the present model.

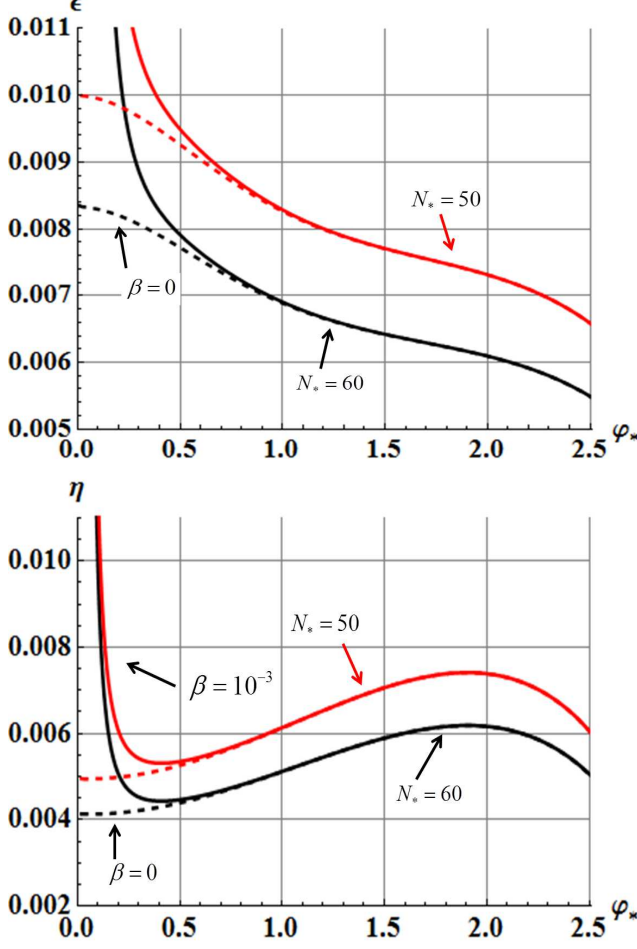


FIG. 1. The slow-roll parameter  $\epsilon$  and  $\eta$  with  $\beta = 0$  (dashed line) and  $\beta = 10^{-3}$  (solid line).

As with a single-field, slow-roll inflation model in Einstein gravity, on sufficiently large scales, we find that the growth of scalar fluctuations depend on two parameters,  $|\dot{\phi}|$  and the Hubble scale  $H$ ; more specifically,  $\mathcal{P}_{\text{sca}}^{1/2} \simeq H^2/(2\pi|\dot{\phi}|)^\dagger$ . The Hubble scale  $H$  is given by (2.1). During a slow-roll

<sup>†</sup> For cosmology based on the Lagrangian (1.4), almost all contributions to primordial scalar fluctuations come from the four-dimensional inflation field mainly because there are no Kaluza-Klein excitations having a mass between  $m^2 = 0$  and  $m^2 = 9H^2/4$  [39, 40]. The massive scalar excitations with mass  $m_{\text{KK}} \geq 3H/2$  are rapidly oscillating and their amplitudes are strongly suppressed on sufficiently large scales [40, 41]. This feature is retained with  $\mathcal{R}^2$  corrections to the Lagrangian at least when  $\sigma \ll V(\phi)$ .

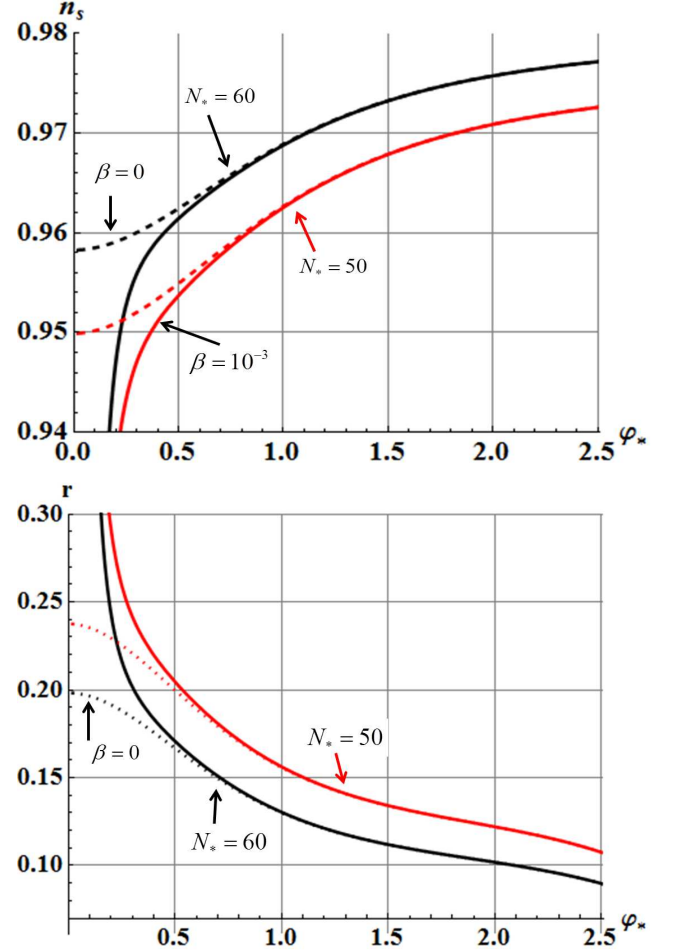


FIG. 2. The scalar spectral index  $n_s$  and tensor-to-scalar ratio  $r$  with  $\beta = 0$  (dashed line) and  $\beta = 10^{-3}$  (solid line).

inflation, since  $\dot{\phi} \simeq -V_\phi/(3H)$ , the amplitude of scalar (density) perturbations is given by [42, 43]

$$A_S^2 \equiv \frac{4}{25} \mathcal{P}_{\text{sca}}(k) \simeq \frac{9}{25\pi^2} \frac{H^6}{V_\phi^2}. \quad (2.14)$$

The normalized amplitude of primordial tensor perturbations is given by [19, 43–45]

$$A_T^2 \equiv \frac{1}{25} \mathcal{P}_{\text{ten}}(k) = \frac{2}{25} \frac{\psi}{M^2 \mathcal{A}} \left( \frac{H}{2\pi} \right)^2, \quad (2.15)$$

$$\mathcal{A} \equiv (1 + \beta) \sqrt{1 + x^2} - (1 - \beta) x^2 \sinh^{-1} \frac{1}{x},$$

where  $x \equiv H/(\psi M) = \beta^{-1/2} [(1 - \beta) \cosh \varphi - 1]^{1/2}$  is a dimensionless measure of the Hubble expansion rate. The power of the scalar and tensor primordial spectra can be calculated approximately in the framework of the slow-roll approximation by evaluating the above equations at the value  $\varphi = \varphi_*$ . On the usual assumption that  $H$  is nearly constant



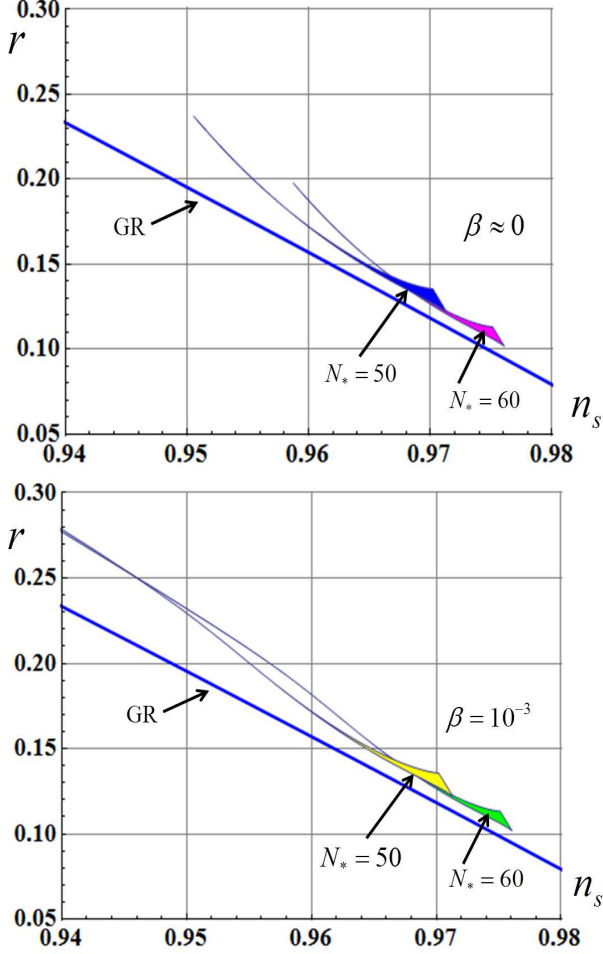


FIG. 3. A parametric plot: The tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  with  $\beta = 0$  (top plot) and  $\beta = 10^{-3}$  (bottom plot).  $\varphi_*$  is varied from  $\varphi_* = 2$  to 0.05. For  $\chi = 0$ , the cosine-form potential approximates to  $m^2\phi^2$  potential [19] and the shaded regions around  $n_s \sim 0.97$  are absent. The single solid line is the prediction of  $m^2\phi^2$  inflation [46] in 4D general relativity.

throughout inflation, the amplitude of scalar density perturbations has some scale dependence due to a small variation in  $V_\phi$ , while the tensor perturbations are roughly scale independent.

The scalar spectral index is given by

$$n_s - 1 \equiv \left. \frac{d \ln A_S^2}{d \ln k} \right|_{k=aH} = -6\epsilon + 2\eta. \quad (2.16)$$

The tensor-to-scalar ratio  $r \equiv 4\mathcal{P}_{\text{ten}}/\mathcal{P}_{\text{sca}}$  is given by

$$r = \frac{16 I(\varphi_*) (1 - \beta)^{3/2} |2\beta|^{1/2} \sinh(3\varphi_*/2) (1 - X(\varphi_*))}{2N_* + 1} \frac{\mathcal{A}}{[(1 - \beta) \cosh \varphi_* - 1]^2}. \quad (2.17)$$

For example, for  $\beta \lesssim 0.003$ ,  $N_* \sim (55 - 58)$  and  $n_s \sim 0.96$  correspond to the values  $\varphi_* \sim (0.570 - 0.425)$  and  $r \sim (0.177 - 0.181)$ . In Figs. 2 and 3 we show the results in wider

ranges,  $\varphi_* \sim 0.05 - 2.5$ ,  $N_* = 50 - 60$ , and  $n_s \sim 0.94 - 0.98$ . The model leads to appropriate values for  $H_*$  and  $V_*$  that are consistent with constraints from Planck data (see below).

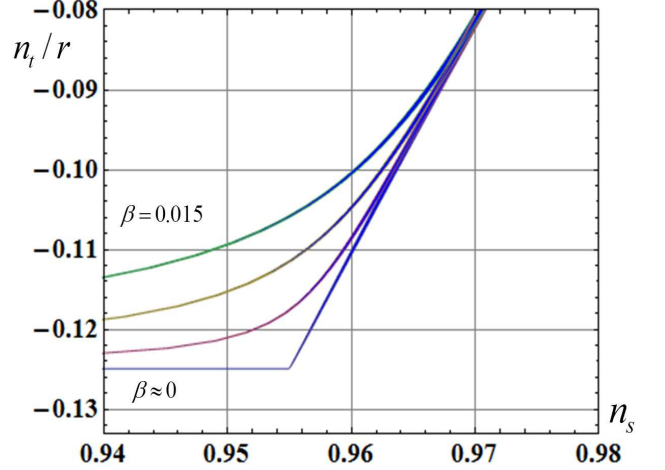


FIG. 4. The ratio  $n_t/r$  versus scalar spectral index  $n_s$  with  $N_* = 55$  and  $\beta = 0.015, 0.005, 0.001$  and  $10^{-7}$  (top to bottom). For  $\beta \approx 0$ , the ratio  $n_t/r$  asymptotes to  $-0.125$  as  $\varphi \rightarrow 0$  (general relativity limit).

As an important consistency check of the model, we compute the tensor spectral index:

$$n_t = \left. \frac{d \ln A_T^2}{d \ln k} \right|_{k=aH} = -2\epsilon \times \frac{1}{\mathcal{A}} \frac{\beta x^2 + \beta + 1}{\sqrt{1 + x^2}}. \quad (2.18)$$

In the limits  $\beta \rightarrow 0$  and  $x \rightarrow 0$ , which means  $H\psi \ll M$ , we recover the standard consistency relation that  $n_t = -2\epsilon$  and  $n_t/r = -1/8$  [47], which relate the tensor spectral index  $n_t$  to the slow-roll parameter  $\epsilon$  and the ratio of the tensor and scalar perturbation amplitudes. For the GB-assisted natural inflation model, we find that the ratio  $n_t/r$  always differs from the result in Einstein gravity. In Fig. 4, we plot the ratio  $n_t/r$  versus the scalar spectral index  $n_s$  for  $N_* = 55$ ; by allowing the coupling constant  $\beta$  in a reasonable range ( $0 \ll \beta < 0.015$ ), we find that the ratio  $n_t/r$  is in between  $-0.1002$  and  $-0.1098$  for  $n_s \approx 0.96$ . This ratio, which only modestly depends on  $N_*$ , is about (13%–24%) less than the value predicted for models based on Einstein gravity. This is one of the testable predictions of the model.

Here we want to make a remark. Measuring  $n_t$  may be challenging with current technologies. However, if  $r > 0.11$  as indicated by the BICEP2 data ( $r = 0.16^{+0.06}_{-0.05}$  after subtracting an estimated foreground), this might be feasible with the next generation of space explorations [48, 49]. Recently, in [50], R. Easther *et al.* found that the ratio  $n_t/r$  is picked around  $-0.15$  for a multifield inflation characterized by the potentials  $V \sim \sum_i \lambda_i |\phi_i|^p$  with  $p > 3/4$ , which differed from the prediction of single-field slow-roll inflation by  $5\sigma$  C.L. This prediction is much larger than for single-field, slow-roll inflation in Einstein gravity; a larger value of  $n_t/r$  usually means a

smaller  $r$ , which seems contradictory to the value of  $r$  reported by the BICEP2 experiment. The model proposed in [50] may be compatible with the BICEP2 data if  $|n_t| \gtrsim 0.024$ . The Planck results put a constraint like  $|n_t| \leq 2\epsilon \lesssim 0.02$ .

### III. OBSERVATIONAL CONSTRAINTS

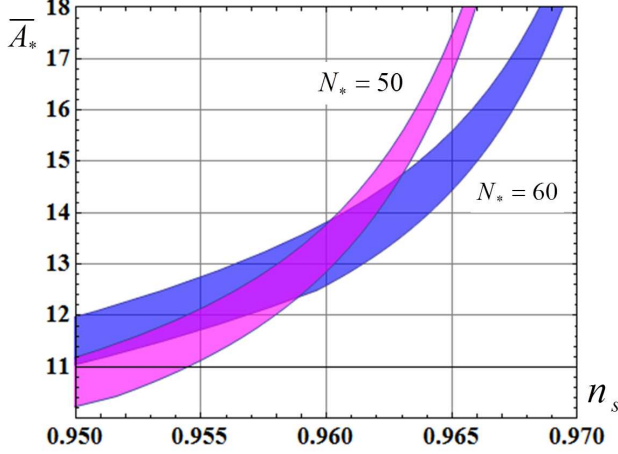


FIG. 5. The COBE normalized amplitude of scalar perturbations  $\bar{A}_* \equiv (M_P/M)^6 \times A_*$  versus  $n_s$  with  $N_* = 50$  and  $N_* = 60$  and  $0 < \beta < 0.01$ .

In order to constrain the model parameters, we use, as in [19], the COBE normalisation for amplitude of scalar perturbations used by the Planck Collaboration [6],  $A_* \simeq V^3/(12\pi^2 M_P^6 V_\phi^2) \simeq 22 \times 10^{-10}$ , where  $A_*$  is approximated as

$$A_* = \frac{2(2N_* + 1)(1 - \beta)^3}{27\pi^2} \frac{M^6}{I(\varphi_*)} \frac{[\sinh(3\varphi_*/2)]^2}{M_P^6 (1 - X(\varphi_*))}, \quad (3.1)$$

where, as usual,  $\varphi_*$  denotes the value of  $\varphi$  at the epoch at which a scale of wave number  $k$  crosses the Hubble radius during inflation. By plotting  $A_*$  versus the scalar spectral index  $n_s$  (shown in Fig. 5), we find that  $n_s \simeq 0.9603$  and  $N_* \sim 55$  correspond to the value

$$A_* \simeq 13.6 \times (M/M_P)^6 \rightarrow M \simeq 0.0233425 \times M_P.$$

By using this result, along with the dimensional reduction relation  $\psi M_P^2 = (1 + \beta)M^2$  [24] between the four- and five-dimensional Planck masses, which holds as long as  $\beta$  and  $\psi$  are constants<sup>‡</sup>, one may express  $\psi$  in terms of  $\beta$  or vice versa.

The BICEP2 data appear to be consistent with the 2013 Planck constrain on the scalar spectral index ( $n_s \simeq 0.96$ ). The COBE normalized number of  $e$ -folds (between the exit of wavelengths now comparable to the observable universe

Table I: The set of parameters that lead to the observationally preferred values of  $n_s \sim 0.96$  and the number of  $e$ -folds  $N \sim 57$ .

$\beta$	$\psi$ ( $10^{-5}$ )	$\varphi_*$	$H_*$ ( $10^{14}$ GeV)	$V_*^{1/4}$ ( $10^{16}$ GeV)
0.015	55.26	0.775	1.39	2.08
0.010	54.99	0.720	1.58	2.13
0.005	54.72	0.643	2.00	2.24
0.003	54.61	0.595	2.39	2.33
0.001	54.49	0.529	3.68	2.58
0.00026	54.46	0.486	6.64	2.98

and the end of inflation) is  $N_{\text{COBE}} \sim 57$  (see below). By taking these two observationally preferred values as input, we estimate in Table I various quantities relevant to inflationary epoch or inflation. This is a set of model parameters that lead to the observationally preferred values of scalar spectral index  $n_s \sim 0.96$  and the number of  $e$ -folds  $N_* \sim 57$ .

The numbers shown in Table I are tentative, which change if  $N_*$  is found to be different from  $N_{\text{COBE}}$ ; if a deviation from  $N_{\text{COBE}}$  is small, then the results are very similar.

We can similarly constrain the model's parameters like  $\Lambda$  and  $f$ . A small curvature coupling as  $\beta \lesssim 0.015$  may be sufficient for suppressing cubic and higher-order curvature corrections in the Lagrangian and also radiative corrections; here we allow  $\beta$  in a slightly wider range  $10^{-6} < \beta < 0.02$ . With  $N_* \simeq 55$  and  $n_s \simeq 0.9603$ , and using the condition (2.11), we observe that

$$21.56 \leq \xi^2 \leq 23.12, \quad \xi^2 \equiv 10^4 \times \frac{4\alpha \times \Lambda^4}{M^2 f^2}. \quad (3.2)$$

The smaller the GB coupling is, the larger the ratio  $\Lambda/\sqrt{Mf}$  would be. If we take the value  $f \sim M_P$  and  $\Lambda \sim 1.0 \times 10^{16}$  GeV  $\equiv \Lambda_*$  as motivated in string-theory models [11, 12] or by CMB observations [6, 7], then we find that  $\alpha \sim 1024 - 1098$  or vice versa.

Of course, the bound (3.2) alters once the number of  $e$ -folds is changed; specifically, with  $N_* = 50 - 60$  and  $n_s = 0.96$ , we have  $4 < \xi^2 < 60$ . Similarly, a deviation from  $n_s \simeq 0.96$  also changes the bound. For  $n_s = 0.9603 \pm 0.0073$  (which is within 68% or 1 $\sigma$  confidence level result of Planck 2013 data) and  $N_* = 50 - 60$ , the bound on  $\xi^2$  is given by

$$3 < \xi^2 < 150. \quad (3.3)$$

If  $\beta$  is closer to zero then  $\xi^2$  is closer to the lower limit. If  $\alpha \simeq 0$  then one would require a much larger value for  $\Lambda$  that is inconsistent with an upper bound on the energy scale of inflation,  $V_*^{1/4} = 1.94 \times 10^{16}$  GeV  $(r_*/0.12)^{1/4}$  [6]. The  $\alpha = 0$  case is ruled out; inflation based on Randall-Sundrum cosmology [38, 51] cannot explain the observational bound on the energy scale of inflation. One would require  $\beta > 0.0001$  (and hence  $\alpha \gtrsim 180$ ) for consistency of the model with Planck results. Indeed, in the present context, the value of the Gauss-Bonnet coupling is very important for a determination of the energy scale of cosmic inflation or vice versa.

In Fig. 6, we plot the function  $\xi^2$  by varying the GB coupling in the range  $\alpha \sim (10^2 - 10^4)$ . For a larger  $\alpha$ , the ratio  $\Theta$

<sup>‡</sup> During inflation, since  $\mathcal{E}/a_0^4 \approx 0$ ,  $\beta$  and  $\psi$  are constants.

is smaller. As the plot shows, the value of  $\Theta$  may be allowed anywhere between 1 and 100, which translates to the bound

$$0.006132 < \Lambda/\sqrt{M_P f} < 0.01. \quad (3.4)$$

For  $f \sim 0.68 \times M_P$ , as motivated in string-theory models, we find that  $\Lambda \sim (0.63 - 2.0) \times 10^{16}$  GeV. This bound is fully consistent with an upper limit on the energy scale of inflation from Planck data  $V_*^{1/4} \lesssim 2.37 \times 10^{16}$  GeV (for  $r_* < 0.27$ ) or  $\Lambda < 2.17 \times 10^{16}$  GeV. The smaller is the value of  $f$  (with  $f < M_P$ ), the narrower would be the bound for  $\Lambda$ , which is desirable both theoretically and observationally. A similar bound on  $V_*^{1/4}$ , namely  $V_*^{1/4} \sim (2.07 - 2.40) \times 10^{16}$  GeV, was obtained in [52] imposing that  $r_* \sim 0.15 - 0.27$ .

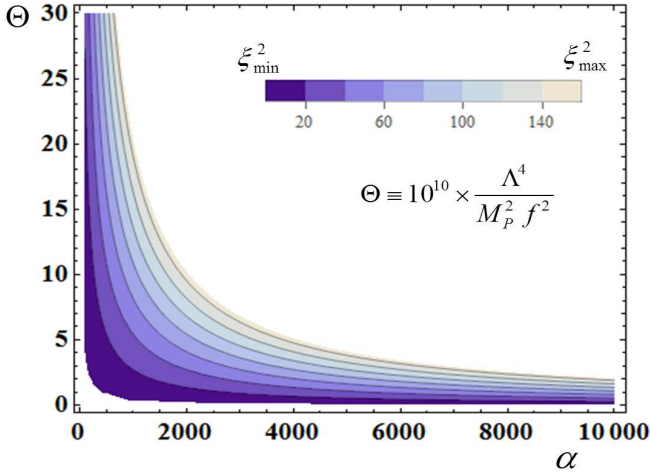


FIG. 6. The bound  $3 < \xi^2 < 150$  as a function of  $\Theta$  and the GB coupling  $\alpha$ .

Note that  $R \propto H^2$  and  $\mathcal{R}^2 \propto H^4$ . This implies, for example, if  $\alpha \sim 10^4$ , the Gauss-Bonnet term  $\alpha(\mathcal{R}^2/M^2)$  is subleading to the Einstein-Hilbert term for  $H/M < 10^{-2}$ . In fact, the Planck data put an upper bound on the Hubble scale of inflation, namely,  $H_* < 8.8 \times 10^{14}$  GeV. So, with  $M \sim 5.67 \times 10^{16}$  GeV, the  $\mathcal{R}^2$  term is subleading to the Einstein-Hilbert term for  $\alpha \lesssim 5 \times 10^3$ . A larger  $\alpha$  than this may be allowed if  $H < H_*$ .

The recent analysis of Planck+WAMP-9+high L+BICEP2 data leads to slightly modified bounds, namely  $V_*^{1/4} = 2.4 \times 10^{16}$  GeV  $(0.27/r(k_*))^{1/4}$  and  $0.15 < r(k_*) < 0.27$  at the pivot scale,  $k_* = 0.002 \text{ Mpc}^{-1}$ .

The above estimate is only tentative since the results depend on the ultimate values of  $N_*$  and  $n_s$ . Nevertheless, the numbers are quite impressive in the sense the GB-assisted “natural inflation” is in perfect agreement with Planck data for a reasonable range of the energy scale of inflation, number of  $e$ -folds and scalar spectral index. The observation that GB-assisted natural inflation parametrized by the potential (1.2) is consistent with the Planck bound on the energy

scale of inflation  $V_*^{1/4}$  and also with the recent BICEP result  $r_* = 0.19^{+0.007}_{-0.005}$  with  $f \lesssim M_P$  is quite remarkable. For values of  $f$  sufficiently near  $M_P$ , sufficient inflation takes place for a broad range of initial values of the field  $\phi$ .

#### Limits on $V_*^{1/4}$ and shift in $\phi$

For  $N_* \sim 55$ , the scalar spectral index  $n_s \sim 0.9603^{+0.005}_{-0.005}$  corresponds to the tensor-to-scalar ratio to  $r_* = 0.176^{+0.028}_{-0.039}$  and to the field value  $\varphi_* \sim 0.31 - 1.10$ . As shown in Figure 7 the variation  $\varphi_* \sim (0.31 - 1.1)$  implies  $\varphi_e \sim (0.03 - 0.12)$ . It follows that  $\Delta\varphi \equiv \varphi_* - \varphi_{\text{end}}$ , the change in  $\varphi$  after the scale  $k_*$  leaves the horizon,  $\Delta\varphi \sim (0.28 - 0.98)$ , depending upon the energy scale of the inflation. This implies

$$V_{\text{end}}^{1/4} \sim 0.55 \times V_*^{1/4}. \quad (3.5)$$

The slow-roll condition is well satisfied, which guarantees the existence of an inflationary epoch. The condition  $V_* \gg \sigma$  is also justified. Thus, if

$$\phi_* \sim \pi f, \quad \text{then} \quad \phi_{\text{end}} \sim 5f/3.$$

Likewise, if  $\phi_*/f \sim 2$  then  $\phi_{\text{end}}/f \sim 1.35$ , which means  $\Delta\phi = \phi_* - \phi_{\text{end}} < 1.47f$ . The distance traversed by the inflaton field  $\phi$  is always sub-Planckian as long as  $f < 0.68 M_P$ , which means the trans-Planckian problem [15, 53, 54] is absent. This is a direct consequence of the fact that the  $\mathcal{R}^2$  corrections ease the slow-roll conditions for inflation and enable inflation to take place at field values below  $M_P$ . This is a very important result in view of the earlier observation (in conventional GR) that the model agrees with Planck+WP data for  $f > 5M_P$  [6]. The above conclusion is qualitatively the same for  $N_* \sim 50 - 60$ .

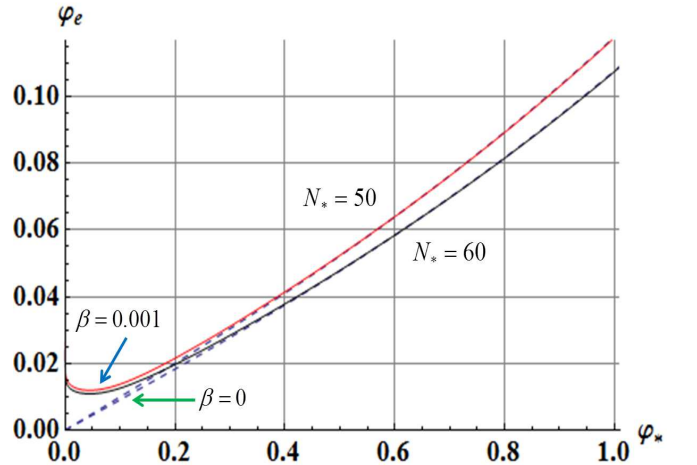


FIG. 7. The  $\varphi_e$  as a function of  $\varphi_*$  for  $\beta = 0$  (dashed lines) and  $\beta = 0.001$  (solid lines).

The above results also apply to GB-assisted  $m^2\phi^2$  inflation (with  $m \equiv \Lambda^2/(\sqrt{2}f)$ ) as it is a limiting case of GB-

assisted natural inflation, especially, around and below the energy scale of inflation,  $\varphi_* \lesssim 1.1$ . The prediction for running scalar spectral index in natural inflation may be different from that in the case of chaotic inflation. Near future observations from Planck experiments for the running spectral index may achieve enough accuracy to allow us to distinguish GB-assisted natural inflation from GB-assisted chaotic inflation.

#### IV. REHEATING OF THE UNIVERSE

Once  $\phi$  rolls (roughly) below  $0.1 f$ , or when  $\varphi \ll \varphi_{\text{end}}$ , the field evolution may be described in terms of oscillations about the potential minimum. For small enough amplitude, the potential is well approximated by  $V(\phi) = m^2 \phi^2$  with  $m^2 \equiv (\Lambda^4/2f^2) \sim (9.6 \times 10^{13} \text{ GeV})^2$  for  $\Lambda \sim 1.5 \times 10^{16} \text{ GeV}$  and  $f \sim 1.65 \times 10^{18} \text{ GeV}$  (to be roughly consistent with the normalization of the power spectrum discussed in the above section).

As in *natural inflation* and  $m^2 \phi^2$ -inflation scenarios in Einstein gravity, at the end of the slow-rolling regime, the field  $\phi$  oscillates about the minimum of the potential and gives rise to particle and entropy production. The cold inflaton-dominated universe can undergo a phase of reheating once the field value drops well below  $0.1 M_P$ , during which the inflaton decays into ordinary particles and the Universe becomes radiation dominated. The reheating temperature may be approximated by

$$T_{\text{RH}} \sim V_{\text{end}}^{1/4} \left( \frac{\Gamma}{M} \right)^{1/2} \sim \left( \frac{45}{4\pi^2 g_*} \right)^{1/4} (\Gamma M_P)^{1/2} \quad (4.1)$$

where  $\Gamma$  is the decay rate of the  $\phi$  field into light fermions (or gauge bosons) and  $g_*$  is the number of relativistic degrees of freedom. In the above result we used the approximation  $V_{\text{end}} \sim 4 \times 10^{-2} M^4 / \sqrt{8\alpha}$ ,  $M \sim 0.0233 M_P$ , and  $\sqrt{\alpha} \sim 0.08 \times g_*$ , so that it closely resembles with the result obtained by Adams *et.al.* in [12]. Here we are only trying to make a rough estimate of  $T_{\text{RH}}$ , so the precise value of  $g_*$  or the GB coupling does not make a big difference to any of the statements below. On dimensional grounds, the decay rate is given by (see, for example, [55])

$$\Gamma \simeq \Upsilon^2 \times m_\phi,$$

where  $\Upsilon \equiv g \times (m_\phi/f)$  is the Yukawa coupling and  $g$  is an effective coupling constant. This approximation is valid not just in a Minkowski space but also in an expanding universe [56], provided that  $H \ll m_\phi$  (during reheating). Hence

$$\Gamma \simeq g^2 \frac{m_\phi^3}{f^2} = g^2 \frac{\Lambda^6}{f^5}, \quad (4.2)$$

where we used  $m_\phi^2 \equiv V_{\phi\phi}|_{\phi=0} = \Lambda^4/f^2$ . Equation (4.1) reads as

$$T_{\text{RH}} \sim \frac{0.35}{\alpha^{1/8}} \times \left( \frac{M}{f} \right)^{1/2} \frac{g \Lambda^3}{f^4}. \quad (4.3)$$

For example, if we take  $f \sim 0.6 M_P$  and  $\Lambda = 1.51 \times 10^{16} \text{ GeV}$  [which is well inside the bound defined by Eq. (3.4)] and  $g \sim 0.1$ , then tentatively we find that  $T_{\text{RH}} \simeq 1.1 \times 10^{10} \times \alpha^{-1/8} \text{ GeV}$ . For example, for  $\alpha \sim 10^2$ , this yields  $T_{\text{RH}} \sim 6.2 \times 10^9 \text{ GeV}$ , which is physically viable.

The number of  $e$ -folds between the exit of wavelengths now comparable to the observable universe and the end of inflation, or the COBE normalized number of  $e$ -folds is

$$N_{\text{COBE}} \sim 62 - \ln \frac{10^{16} \text{ GeV}}{V_*^{1/4}} + \ln \frac{V_*^{1/4}}{V_{\text{end}}^{1/4}} - \frac{1}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_{\text{RH}}^{1/4}}, \quad (4.4)$$

where  $\rho_{\text{RH}}$  is the energy density in radiation as a result of reheating. If  $\rho_{\text{RH}}^{1/4} \sim T_{\text{RH}}$ , then using (3.5) we find  $N_{\text{COBE}} \sim 57$ . The spectral index approximated by  $n_s \sim 1 - 2/N_{\text{COBE}}$  is  $n_s \sim 0.965$  – a value which is well within  $1\sigma$  confidence level (68%) of the Planck data. This shows the consistency of the model, independent of a bound on  $r$ .

A theory of baryogenesis during reheating discussed, for example, in [57, 58] (see also [59, 60]) to explain how particle production after the end of inflation can be applied to the present model. As in the standard natural inflation model, baryogenesis can take place mostly during the reheating era, whereas nucleosynthesis can take place at a later stage but well before the Universe enters into a late-epoch cosmic acceleration – the second epoch of cosmic inflation but at a much slower pace. The inflaton density can drop significantly after a period of parametric resonance (or during the phase of coherent oscillations). The Universe can decelerate all the way until  $\rho^{1/4}$  drops below 238 GeV. A detailed theory of baryogenesis would require a deeper understanding of particle physics around the energy scale of reheating, such as, the effects of various interactions between  $\phi$  and fermions and the other decay products of  $\phi$  (or bosons). This topic is beyond the scope of this paper.

#### V. DARK ENERGY COSMOLOGY

In this section we establish that the model may be used to explain the concurrent universe with the right amount of dark energy equation of state and the present Hubble scale.

##### A. Low energy limit

At low energies,  $\varphi = \varphi_0 \ll 1$ . Expanding around  $\varphi_0 = 0$ , we find that

$$H^2 = \frac{M^2 \psi_0^2}{\beta_0} \left[ (1 - \beta_0) \left( 1 + \frac{\varphi_0^2}{2} + \dots \right) - 1 \right], \quad (5.1)$$

$$\varphi_0 \approx \frac{2(\rho + \sigma)}{3\psi_0 M^4} \frac{(2\beta_0)^{1/2}}{4(1 - \beta_0)^{3/2}}, \quad (5.2)$$

where

$$\beta_0 \equiv 4\alpha\psi_0^2 = 1 - \left( 1 + 8\lambda\alpha + \frac{8\alpha\mathcal{E}}{a_0^4 M^2} \right)^{1/2}. \quad (5.3)$$



In principle,  $\rho = \rho_M + \rho_R + \rho_\phi$  but the inflaton contribution can be negligibly small at late epochs since  $\phi = 0$  is a minimum for a cosine-form potential, which means  $\rho \simeq \rho_M + \rho_R$ . After inflation (more precisely, after reheating), the bulk radiation term proportional to  $\mathcal{E}(a_0)$  is nonzero. The GB coupling  $\alpha = \beta_0/(4\psi_0^2)$  is assumed to be a constant.<sup>§</sup>

For  $\rho + \sigma \ll M^4$  and  $\beta_0 \ll 1$ , the Friedmann equation reduces to

$$H^2 = H_0^2 + \frac{\rho^2}{36(1-\beta_0)^2 M^6} + \frac{\sigma\rho}{18(1-\beta_0)^2 M^6}, \quad (5.4)$$

where

$$\begin{aligned} H_0^2 &\equiv -M^2\psi_0^2 + \frac{\sigma^2}{36(1-\beta_0)^2 M^6} \\ &\simeq -M^2\psi_0^2 + \frac{\sigma^2}{36M^6} + \frac{\beta_0\sigma^2}{18M^6}. \end{aligned} \quad (5.5)$$

On large scales,  $cH_0^{-1} \sim 1.3 \times 10^{28} \text{ cm} \sim 4222 \text{ Mpc}$ , the proportion of dark energy and (ordinary plus dark) matter appear to be 68.3% and 31.7% at present, which means

$$\begin{aligned} \Omega_m &= \frac{\sigma\rho}{18(1-\beta_0)M^6 H^2} \left(1 + \frac{\rho}{2\sigma}\right) \sim 0.317, \\ \Omega_\Lambda &\equiv \frac{H_0^2}{H^2} \sim 0.683. \end{aligned} \quad (5.6)$$

Moreover,  $\rho \gtrsim \rho_c = 3.98 \times 10^{-47} \text{ GeV}^4$ , which means  $\sigma^{1/4} \lesssim 1.09 \times 10^{16} \text{ GeV}$ . This is not surprising because the brane tension is large when the size of the Universe is also large, which is actually proportional to the volume of the Universe,<sup>¶</sup>. There also exists a lower bound on the 3-brane tension (see below). Here we must note that  $\sigma$  is a free parameter and the RS-type fine-tuning of brane tension,  $\sigma = 2M^4\psi_0(3-\beta_0) \sim 6M^4\psi_0$  (since  $\beta_0 \simeq 0$ ) holds only when  $\rho = 0$ ,  $H = 0$  and also  $\mathcal{E} = 0$ , but *not* if any of these quantities is not zero.

As in the  $\alpha = 0$  case [62], the Universe can undergo transition from decelerating to accelerating expansion when

$$w_{\text{eff}} = \frac{p-\sigma}{\rho+\sigma} = \frac{w-\xi}{1+\xi} \lesssim -1/3, \quad (5.7)$$

where  $\xi \equiv \sigma/\rho$  and  $w = p/\rho$  is the equation of state of matter or radiation. For example, for  $\xi = 200$ , we obtain  $w_{\text{eff}} \simeq -0.995$ , which is indistinguishable from the effect of a pure cosmological constant. Acceleration kicks in first on the largest scales as the condition  $\rho \ll \sigma$  is achieved there at first. It should be noted that the condition  $\rho < 2\sigma$  is *not* always

sufficient for the occurrence of cosmic acceleration; it also depends on the relative ratio  $\rho_\Lambda/\rho$  or the ratio  $\nu \equiv \sigma/(6H_0M^3)$  (in the present model). For example, a domain of spacetime with  $\nu \gg 1$  does not enter into an accelerating phase unless that  $\xi \gg 1$  is attained.

## B. Late epoch acceleration

In the post-inflationary universe it is natural to assume that the energy density decays as

$$\rho = \frac{\rho_*}{a^\gamma}, \quad \gamma = 3(1+w), \quad (5.8)$$

where  $\rho_*$  is a constant and  $\gamma = 3$  ( $\gamma = 4$ ) for ordinary matter (radiation). Equation (5.4) admits an exact solution, which is given by

$$\begin{aligned} a^\gamma &= \frac{\rho_* \nu}{\sigma(1-\beta_0)^2} \\ &\times \left[ (1-\beta_0) \sinh(\gamma H_0 t) + \nu (\cosh(\gamma H_0 t) - 1) \right], \end{aligned} \quad (5.9)$$

where  $\nu \equiv \sigma/(6H_0M^3)$ . The matter (radiation) density evolves as

$$\rho = \left(\frac{\sigma}{\nu}\right) \frac{(1-\beta_0)^2}{(1-\beta_0) \sinh(\gamma H_0 t) + \nu (\cosh(\gamma H_0 t) - 1)}. \quad (5.10)$$

In the limit  $\beta_0 \rightarrow 0$ , we recover the results in [61, 62]. The Hubble expansion parameter and deceleration parameters are obtained by using the definition  $H := \dot{a}/a$  and  $q = -1 - \dot{H}/H^2$ . In fact,  $H_0 = 0$  is *not* a physical choice, so we take  $H_0 > 0$ . It is readily seen that the scale factor grows in the beginning as  $t^{1/\gamma}$  but at a late epoch it grows almost exponentially,

$$\begin{aligned} a(t) &\simeq \left( \frac{\rho_* \nu}{2\sigma(1-\beta_0)^2} \right)^{1/\gamma} \left( (1-\beta_0 + \nu) e^{\gamma H_0 t} - 2\nu \right)^{1/\gamma} \\ &= \left( \frac{\rho_*}{\sigma} \right)^{1/\gamma} \left( e^{\gamma H_0 t} - 1 \right)^{1/\gamma}, \end{aligned} \quad (5.11)$$

where the equality holds in the limit  $\nu \rightarrow 1$  and  $\beta_0 \rightarrow 0$ . The result shows that after the end of inflation (more precisely, after reheating) the scale factor could grow much slower than that predicted by Einstein gravity; specifically,  $a \propto t^{1/4}$  ( $\propto t^{1/3}$ ) during radiation (matter) dominated era. The period of structure formation can be longer than in GR.

In Figs. 8 and 9, we show a parametric plot between  $q$  (deceleration parameter) and the ratio  $\rho/\sigma$ . The period of deceleration prior to the late-epoch acceleration becomes longer for  $\nu$  larger than unity; the deceleration of the Universe is also slower (as compared to the  $\nu = 1$  case). The Universe enters into an accelerating phase at a relatively late time if  $\nu \gg 1$ . The above result reveals a genuine possibility of realizing four-dimensional cosmology for which the Universe decelerates between the two periods of cosmic acceleration,

<sup>§</sup> As a variant of this idea, one may allow a slowly varying  $\alpha$  as in string theory where  $\alpha$  is proportional to the Regge slope  $e^{\tilde{\phi}}/g_s^2$ , where  $\tilde{\phi}$  is the dilaton and  $g_s$  is the string coupling.

<sup>¶</sup> If we take a smaller patch of the Universe, then the brane tension is also small. For example, on galactic distances, the value of  $\sigma$  can be much smaller than its value on Hubble scales.

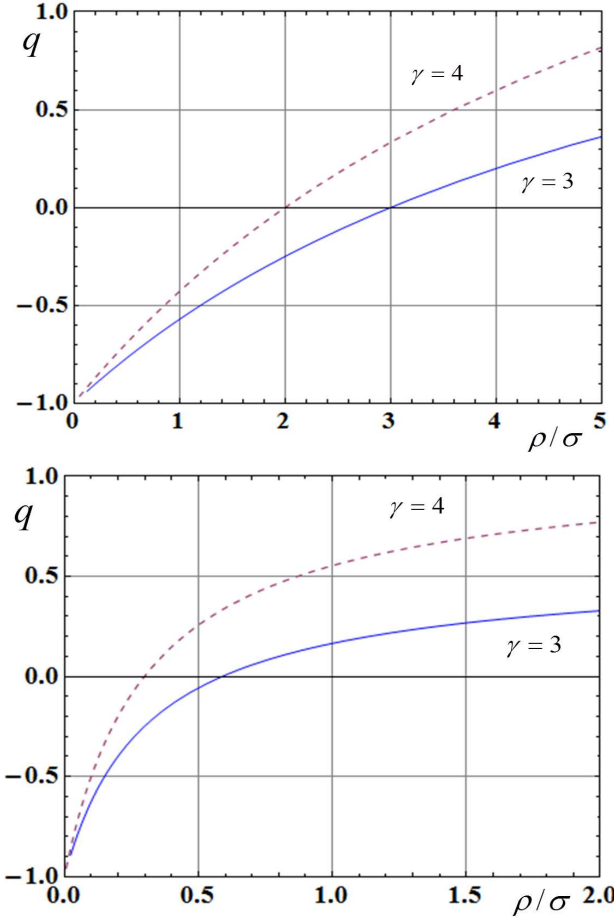


FIG. 8. A parametric plot: Deceleration parameter  $q$  versus the ratio  $\rho/\sigma$  with  $\nu = 1$  (top plot) and  $\nu = 10$  (bottom plot).  $H_0 t$  is varied from 0.01 to 1.5 and  $\beta_0 \gtrsim 0$ .

i.e. between the primordial inflation and the late-epoch acceleration at a much lower energy scale [63, 64].

The positivity energy condition ( $\rho > 0$ ) plus the condition  $H_0^2 \gtrsim 0$  implies that

$$\nu \equiv \frac{\sigma}{6H_0 M^3} \gtrsim 1. \quad (5.12)$$

$H_0$  may be taken to be the present Hubble scale  $\bar{H}_0 = 2.1332h_0 \times 10^{-42} \text{ GeV} \sim 1.5 \times 10^{-42} \text{ GeV}$  on sufficiently large scales ( $h_0 \simeq 0.71$  following [65]) \*\* Hence

$$\sigma^{1/4} \gtrsim 200.45 \text{ GeV}. \quad (5.13)$$

In fact, the condition  $\nu \gtrsim 1$  also implies

$$8\alpha H_0^2 \ll M^2, \quad \beta_0 = 4\alpha\psi_0^2 \simeq 0 \quad (5.14)$$

\*\* This fine-tuning may be taken as a restatement in the brane-world scenario of the cosmological constant problem and we do not attempt to solve it here.

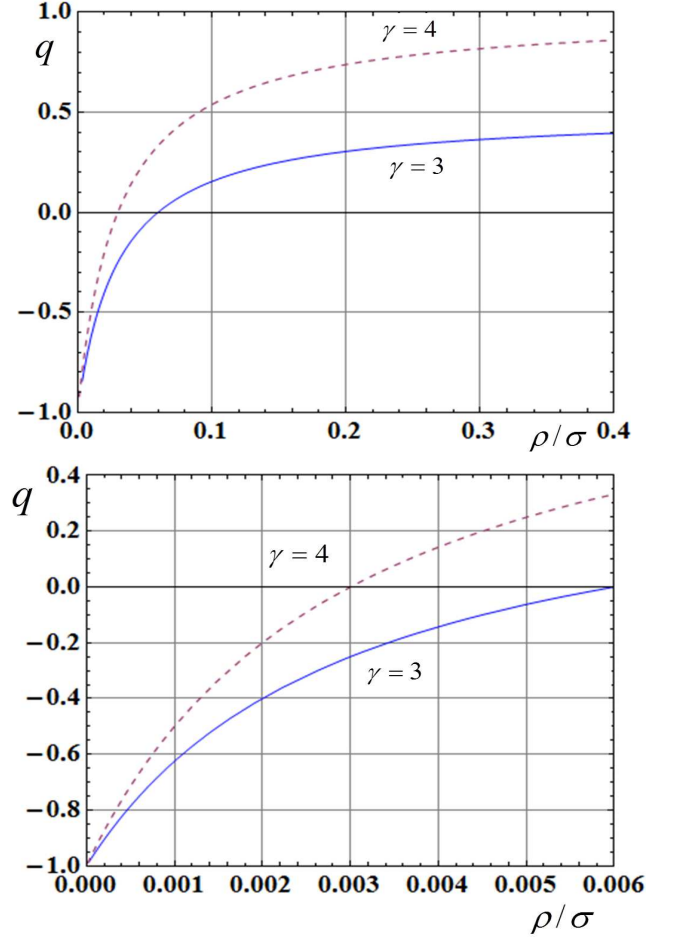


FIG. 9. As in Fig. 8 but now  $\nu = 10^2$  (top plot) and  $\nu = 10^3$  (bottom plot).

(on sufficiently large scales) which are obviously always satisfied [cf. Eq. (5.5)]. The last condition  $\beta_0 \simeq 0$  further implies that

$$-\lambda M^2 \simeq \frac{\mathcal{E}}{a_0^4} \quad (5.15)$$

to a large accuracy. This result is *not* unnatural though – the cosmic expansion of our Universe could naturally take us into a state of equilibrium where the bulk cosmological constant  $(-\Lambda_5/3) \equiv \lambda M^2$  in five dimensions equals the contribution of the radiation energy from the bulk. This is also a manifestation of AdS-gravity/Friedmann-Lemaître-Robertson-Walker cosmology correspondence or AdS holography. In the limit  $\lambda \rightarrow 0$ , the bulk spacetime is Minkowski flat, which means  $\mathcal{E} = 0$ . The bulk radiation term is a measure of Weyl curvature which must vanish if  $\lambda = 0$ .

A 3-brane tension of the order of  $(200\text{GeV})^4$  is in minimum range and nucleosynthesis bounds are satisfied even for a low value, such as  $\sigma > (100 \text{ MeV})^4$  [60, 62]. The observed cosmic acceleration of the Universe may *not* be a recent phenomena, which could have rather kicked in when

$\rho \simeq \rho_M + \rho_R < (238 \text{ GeV})^4$ , which means  $\rho^{1/4}$  is already  $\sim 10^{13}$  times less density than the energy scale at the end of inflation.

Here we make one more remark. At a late epoch the effects of the GB term (or the  $\mathcal{R}^2$  corrections) is negligibly small; the model is indistinguishable from the RS model except that all the bounds found in this paper are nonexistent in RS cosmology. Of course, the  $\mathcal{R}^2$ -type corrections are important at the earliest epoch, whose contribution diminishes rapidly after inflation (more precisely, after reheating) all the way to the epochs of baryogenesis, nucleosynthesis, and at the present epoch. This can be understood also by looking at the Lagrangian: at late epochs (and on sufficiently large scales)  $\alpha \mathcal{R}^2/M^2 \propto \alpha H_0^2(H_0^2/M^2) \ll H_0^2$ , while the contribution of the Einstein-Hilbert term  $R \propto H_0^2$ .

## VI. CONCLUSION

The evidence of a direct detection of the primordial “B-mode” polarization of the CMB by BICEP2 telescope [7], with a relatively large tensor-to-scalar ratio  $r \sim 0.19 (+0.007 - 0.005)$  (or  $r = 0.16_{-0.05}^{+0.06}$  after subtracting an estimated foreground), may be viewed as a cosmological gravitational wave signature of primordial inflation. A large value of  $r$ , along with a large value of the energy scale of inflation,  $V_*^{1/4} \sim 2 \times 10^{16} \text{ GeV}$ , naturally point to some modification of Einstein gravity at a scale relevant to inflation. In this paper, for the first time in the literature, we identified a concrete gravitational theory where inflation has natural beginning and natural ending. Inflation is driven by a cosine-form potential,  $V(\phi) = \Lambda^4(1 - \cos(\phi/f))$ . The effect of the  $\mathcal{R}^2$ -terms on the magnitudes of scalar and tensor fluctuations and spectral indices are shown to be important at the energy scale of inflation. The model is trustworthy since a variation of the inflaton field can be smaller than the reduced Planck mass  $M_P$ . The results obtained in this paper are available also for GB assisted  $m^2\phi^2$  inflation [with  $m \equiv \Lambda^2/(\sqrt{2}f) \sim 1.0 \times 10^{14} \text{ GeV}$ ] as it is a limiting case of GB-assisted natural inflation at a slightly lower energy scale than  $V_*^{1/4}$ .

The GB-assisted natural inflation is in agreement with Planck data for a wide range of the energy scales for inflation and the number of  $e$ -folds. The model generates a suppression in scalar power at large scales along with reasonable amplitudes of primordial scalar and tensor perturbations. The GB coupling constant in the range  $\alpha \sim (10^2 - 10^4)$  can lead to observationally preferred values, such as,  $n_s = 0.9603 \pm 0.005$  and  $r \sim 0.14 - 0.21$ ; the latter bound is compatible with the BICEP2 result [7]. Another important prediction of the model is that the ratio  $n_T/r$  is about (13%-24%) less than the value predicted for single-field, slow-roll inflation models based in Einstein gravity ( $n_T/r = -0.125$ ); the  $\mathcal{R}^2$ -type corrections in the Lagrangian enhance the ratio of the tensor and scalar perturbation amplitudes and hence lower the ratio  $n_T/r$ . This gives a novel and testable prediction for the GB-assisted natural inflation model.

The model is natural and well motivated in the context of both particle physics and high-scale string-theory models. It is compatible with CMB data from Planck and BICEP2 experiments as well as low red-shift data from type I supernovae. The latter provides a direct observational evidence for an accelerating expansion of the Universe [63]. So, it may be the correct description of both the early universe cosmology and concurrent universe undergoing an extremely slow accelerating phase in the last few billion years. For the first time in the literature, we have presented a concrete model whose model parameters are found in a narrow range that are consistent with broad theoretical ideas and cosmological constraints from CMB observations by the BICEP2 and Planck telescopes.

A very recent paper from the Planck Collaboration [66] [Planck intermediate results. XXX] appears to show that the BICEP2 gravitational wave result could be due to the dust contamination. This new analysis does not completely rule out BICEP2s original claim just yet – detailed cross-correlation studies of Planck and BICEP2 data would be required for a definitive answer. Nevertheless, the results in this paper are purely theoretical and they are natural outcomes of a “natural inflation model” that takes into account the contributions of  $\mathcal{R}^2$  terms in the Lagrangian, which is separately well motivated. In fact, the model can still satisfy the Planck constraint  $r < 0.13$  provided that the scalar spectrum spectral index is in high end of the  $1\sigma$  result, namely,  $n_s \gtrsim 0.967$ , and/or the number of  $e$ -folds  $N_* \gtrsim 60$ .

Cosmological observations when interpreted in terms of a FLRW metric with (assumed) scale-free density perturbations imply that  $\Lambda_4 \simeq 3H_0^2$ . This is then interpreted as dark energy with  $\rho_{DE} \simeq 3M_P^2 H_0^2$ . Dark energy is a difficult problem in cosmology (see, [67, 68] for reviews on dark energy theory) mainly because it requires setting the key parameter(s) to be of order  $H_0^2$  by hand. Explaining dark energy problem usually means (i) getting the correct equation of state, (ii) getting the right proportion of dark energy and matter (ordinary plus dark), (iii) explaining the triple cosmic coincidence ( $\rho_\Lambda \sim \rho_M \sim \rho_R$ ) around the onset of late-epoch cosmic acceleration, and finally (iv) getting the Hubble scale that asymptotes to  $H_0$  when  $\rho$  gets close to the critical density  $\rho_c$ . These are *not* independent though – each one of these characteristics of the “dark energy” problem follows simply because  $H_0$  is a key physical parameter. In this paper we have shown that instead of picking  $H_0$  by hand we can relate it with the 3-brane tension and the curvature coupling parameters.

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